Exercises  Section 1.5 – Work and Fluid Forces

1. It took 1800 J of work to stretch a spring from its natural length of 2 m to a length of 5 m. Find the spring’s force constant.

2. A spring has a natural length of 10 in. An 800-lb force stretches the spring to 14 in.
   a) Find the force constant.
   b) How much work is done in stretching the spring from 10 in to 12 in?
   c) How far beyond its natural length will a 1600-lb force stretch the spring?

3. It takes a force of 21,714 lb to compress a coil spring assembly on a Transit Authority subway car from its free height of 8 in. to its fully compressed height of 5 in.
   a) What is the assembly’s force constant?
   b) How much work does it take to compress the assembly the first half inch? The second half inch? Answer to the nearest in./lb.

4. A mountain climber is about to haul up a 50 m length of hanging rope. How much work will it take if the rope weighs 0.624 N/m?

5. A bag of sand originally weighing 144 lb was lifted a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been lifted 10 18 ft. How much work was done lifting the sand this far? (Neglect the weight of the bag and lifting equipment.)

6. An electric elevator with a motor at the top has a multistrand cable weighing 4.5 lb/ft. When the car is at the first floor, 180 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. How much work does the motor do just lifting the cable when it takes the car from the first floor to the top?

7. When a particle of mass \( m \) is at \((x, 0)\), it is attracted toward the origin with a force whose magnitude is \( \frac{k}{x^2} \). If the particle starts from rest at \( x = b \) and is acted on by no other forces, find the work done on it by the time reaches \( x = a, 0 < a < b \).
8. The rectangular cistern (storage tank for rainwater) shown has its top 10 ft below ground level. The cistern, currently full, is to be emptied for inspection by pumping its contents to ground level.

a) How much work will it take to empty the cistern?

b) How long will it take a 1/2-hp pump, rated at 275 ft-lb/sec, to pump the tank dry?

c) How long will it take the pump in part (b) to empty the tank halfway? (It will be less than half the time required to empty the tank completely)

d) What are the answers to parts (a) through (c) in a location where water weighs 62.6 \( \frac{lb}{ft^3} \) ? 62.59 \( \frac{lb}{ft^3} \) ?
Solution Section 1.5 – Work and Fluid Forces

Exercise

It took 1800 J of work to stretch a spring from its natural length of 2 m to a length of 5 m. Find the spring’s force constant

Solution

\[ W = \int_{0}^{3} F(x) \, dx \]

\[ 1800 = \int_{0}^{3} kx \, dx \]

\[ 1800 = \frac{1}{2}k x^2 \bigg|_{0}^{3} \]

\[ 1800 = \frac{1}{2}k (9 - 0) \]

\[ 1800 = \frac{9}{2}k \]

\[ 1800 \left( \frac{2}{9} \right) = k \]

\[ k = 400 \frac{N}{m} \]

Exercise

A spring has a natural length of 10 in. An 800-lb force stretches the spring to 14 in.

a) Find the force constant.

b) How much work is done in stretching the spring from 10 in to 12 in?

c) How far beyond its natural length will a 1600-lb force stretch the spring?

Solution

a) \[ k = \frac{F}{x} = \frac{800}{14 - 10} = \frac{800}{4} \]

\[ k = 200 \frac{lb}{in} \]

b) \[ \Delta x = 12 - 10 = 2 \text{ in} \]

\[ W = k \int_{0}^{2} x \, dx \]

\[ = 200 \frac{1}{2} x^2 \bigg|_{0}^{2} \]

\[ = 200 \frac{1}{2} \cdot 2^2 \]

\[ = 400 \text{ ft-lb} \]


\[
\begin{align*}
= 100(4 - 0) \\
= 400 \text{ in.lb} \\
= 400 \frac{1 \text{ ft}}{12 \text{ in}} \text{ in.lb} \\
= 33.3 \text{ ft} \cdot \text{lb}
\end{align*}
\]

c) \( F = 200x \)
\[
1600 = 200x \\
\frac{1600}{200} = x \\
x = 8 \text{ in}
\]

**Exercise**

It takes a force of 21,714 lb to compress a coil spring assembly on a Transit Authority subway car from its free height of 8 in. to its fully compressed height of 5 in.

a) What is the assembly’s force constant?

b) How much work does it take to compress the assembly the first half inch? The second half inch? Answer to the nearest in/-lb.

**Solution**

a) \( F = kx \)
\[
21714 = k(8 - 5) \\
21714 = 3k \\
k = 7238 \text{ lb/in}
\]

b) \( W = k \int_{0}^{0.5} x dx \)
\[
= 7238 \left[ \frac{1}{2} x^2 \right]_{0}^{0.5} \\
= 7238 \left[ \frac{1}{2} (0.5)^2 - 0 \right] \\
= 905 \text{ in} \cdot \text{lb}
\]

\[
W = 7238 \int_{0.5}^{1} x dx \\
= 7238 \left[ \frac{1}{2} x^2 \right]_{0.5}^{1} \\
= 3619 \left[ 1^2 - 0.5^2 \right]_{0.5}^{1} \\
= 2714 \text{ in} \cdot \text{lb}
\]
**Exercise**

A mountain climber is about to haul up a 50 m length of hanging rope. How much work will it take if the rope weighs 0.624 N/m?

**Solution**

\[ W = 0.624 \int_{0}^{50} x \, dx \]

\[ = 0.624 \left[ \frac{1}{2} x^2 \right]_{0}^{50} \]

\[ = \frac{0.624}{2} \left[ (50)^2 - 0 \right] \]

\[ = 780 \text{ J} \]

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**Exercise**

A bag of sand originally weighing 144 lb was lifted a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been lifted to 18 ft. How much work was done lifting the sand this far? (Neglect the weight of the bag and lifting equipment.)

**Solution**

The weight of sands decreases by \( \frac{1}{2} \) 144 = 72 lb over the 18 ft. at rate \( \frac{72}{18} = 4 \text{ lb/f t} \)

\[ F(x) = 144 - 4x \]

\[ W = \int_{0}^{18} (144 - 4x) \, dx \]

\[ = \left[ 144x - 2x^2 \right]_{0}^{18} \]

\[ = 144(18) - 2(18)^2 - (0) \]

\[ = 1944 \text{ ft \cdot lb} \]
**Exercise**

An electric elevator with a motor at the top has a multistrand cable weighing 4.5 lb/ft. When the car is at the first floor, 180 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. How much work does the motor do just lifting the cable when it takes the car from the first floor to the top,?

**Solution**

$$ F(x) = k\Delta x = 4.5(180 - x) $$

$$ W = \int_0^{180} 4.5(180 - x) \, dx $$

$$ = 4.5\left[180x - \frac{1}{2}x^2\right]_0^{180} $$

$$ = 4.5\left[180(180) - \frac{1}{2}(180)^2 - 0\right] $$

$$ = 72,900 \text{ ft} \cdot \text{lb} $$

**Exercise**

When a particle of mass \( m \) is at \( (x, 0) \), it is attracted toward the origin with a force whose magnitude is \( \frac{k}{x^2} \). If the particle starts from rest at \( x = b \) and is acted on by no other forces, find the work done on it by the time reaches \( x = a, 0 < a < b \).

**Solution**

$$ F(x) = -\frac{k}{x^2} $$

$$ W = \int_a^b -\frac{k}{x^2} \, dx $$

$$ = k\int_a^b -\frac{1}{x^2} \, dx $$

$$ = k\left[\frac{1}{x}\right]_a^b $$

$$ = k\left(\frac{1}{b} - \frac{1}{a}\right) $$

$$ = \frac{k(a - b)}{ab} $$
Exercise

The rectangular cistern (storage tank for rainwater) shown has its top 10 ft below ground level. The cistern, currently full, is to be emptied for inspection by pumping its contents to ground level. Assume that the water weighs 62.4 lb/ft³.

a) How much work will it take to empty the cistern?
b) How long will it take a 1/4-hp pump, rated at 275 ft-lb/sec, to pump the tank dry?
c) How long will it take the pump in part (b) to empty the tank halfway? (It will be less than half the time required to empty the tank completely)
d) What are the answers to parts (a) through (c) in a location where water weighs 62.6 lb/ft³?

Solution

a) \( \Delta V = (20)(12)\Delta y = 240\Delta y \)
\[ F = 62.4(\Delta V) = (62.4)240\Delta y = 14976\Delta y \]
\( \Delta W = \text{force} \times \text{distance} = 14976\Delta y \times y \)
\[ W = 14976 \int_{10}^{20} ydy \]
\[ = 14976 \left[ \frac{1}{2} y^2 \right]_{10}^{20} \]
\[ = \frac{14976}{2} \left( 20^2 - 10^2 \right) \]
\[ = 2,246,400 \text{ ft} \cdot \text{lb} \]

b) \[ t = \frac{W}{275 \text{ ft} \cdot \text{lb/sec}} \]
\[ = \frac{2,246,400 \text{ ft} \cdot \text{lb/sec}}{275 \text{ ft} \cdot \text{lb}} \]
\[ \approx 8,168.73 \text{ sec} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \]
\[ \approx 2.27 \text{ hrs} \approx 2 \text{ hrs} \ & \ 16.1 \text{ min} \]

c) \[ W = 14976 \int_{10}^{15} ydy \]
\begin{align*}
&= 14976 \left[ \frac{1}{2} y^2 \right]_{10}^{15} \\
&= 14976 \left( 15^2 - 10^2 \right) \\
&= 936,000 \text{ ft} \cdot \text{lb}
\end{align*}

\begin{align*}
t &= \frac{W}{275 \text{ ft} \cdot \text{lb sec}} \\
&= \frac{936,000 \text{ ft} \cdot \text{lb sec}}{275 \text{ ft} \cdot \text{lb}} \\
&\approx 3403.64 \text{ sec} \frac{1}{60 \text{ sec}} \\
&\approx 56.7 \text{ min}
\end{align*}

d) Water weighs 62.26 lb/ft³

\begin{align*}
W &= (62.26)(240)(150) \\
&= 2,214,360 \text{ ft} \cdot \text{lb}
\end{align*}

\begin{align*}
t &= \frac{W}{275 \text{ ft} \cdot \text{lb sec}} \\
&= \frac{2,214,360 \text{ ft} \cdot \text{lb sec}}{275 \text{ ft} \cdot \text{lb}} \\
&\approx 8,150.4 \text{ sec} \frac{1}{3600 \text{ sec}} \\
&\approx 2.264 \text{ hrs} \quad 2 \text{ hrs} \ & 15.8 \text{ min}
\end{align*}

\begin{align*}
W &= (62.26)(240)\left(\frac{150}{2}\right) \\
&= 933,900 \text{ ft} \cdot \text{lb}
\end{align*}

\begin{align*}
t &= \frac{W}{275 \text{ ft} \cdot \text{lb sec}} \\
&= \frac{933,900 \text{ ft} \cdot \text{lb sec}}{275 \text{ ft} \cdot \text{lb}} \\
&\approx 3396 \text{ sec} \frac{1}{60 \text{ sec}} \\
&\approx 56.6 \text{ min}
\end{align*}

Water weighs 62.59 lb/ft³

\begin{align*}
W &= (62.59)(240)(150) \\
&= 2,253,240 \text{ ft} \cdot \text{lb}
\end{align*}
\[ t = \frac{W}{275 \text{ ft} \cdot \text{lb}} \text{ sec} \]

\[ = \frac{2,253,240 \text{ ft} \cdot \text{lb}}{275} \text{ sec} \]

\[ \approx 8,193.60 \text{ sec} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \]

\[ \approx 2.276 \text{ hrs} \approx 2 \text{ hrs} \& 16.56 \text{ min} \]

\[ W = (62.59)(240)(\frac{150}{2}) \]

\[ = 938,850 \text{ ft} \cdot \text{lb} \]

\[ t = \frac{W}{275 \text{ ft} \cdot \text{lb}} \text{ sec} \]

\[ = \frac{938,850 \text{ ft} \cdot \text{lb}}{275} \text{ sec} \]

\[ \approx 3414 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \]

\[ \approx 56.9 \text{ min} \]