Name_____ Date_____ Period_____

Mixed Integration Worksheet

Part I:
For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int\left(x^3+1\right)dx$ $\int x^2\left(x^3+1\right)^4 dx$ $\int\sqrt{x^3+1}dx$ $\int\left(x^3+1\right)^2 dx$

B. $\int\sqrt{x}\left(1-x^2\right)dx$ $\int\sqrt{1-x^2}dx$ $\int\frac{1}{\sqrt{1-x^2}}dx$ $\int\frac{xdx}{\sqrt{1-x^2}}$

C. $\int\cos^2 x\sin^3 xdx$ $\int\sqrt{1-\cos^2 x}dx$ $\int\frac{dx}{\cos^2 x}$ $\int\frac{dx}{\cos x\sqrt{\sin x}}$

D. $\int \tan x \sec x dx$ $\int \tan x \cos x dx$ $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ $\int \frac{dx}{\tan x + 1}$

E. $\int e^{-x^2} dx$ $\int \frac{e^x}{3 + e^x} dx$ $\int \left(e^x + 3\right) dx$ $\int \frac{\ln\left(e^{2x}\right)}{x^2} dx$

Part II: Evaluate the integrals

1. $\int \left(5x + 4\right)^5 dx$
2. $\int 3t^2 \left(t^3 + 4\right)^5 dt$
3. $\int \sqrt{4x - 5} dx$
4. $\int t^2 \left(t^3 + 4\right)^{-1/2} dt$
5. $\int \cos\left(2x + 1\right) dx$
6. $\int \sin^{10} x \cos x dx$

7. $\int \frac{\sin x}{\cos^5 x}\,dx$

8. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx$

9. $\int \sqrt{x^3+x^2}\left(3x^2+2x\right)dx$

10. $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx$

11. $\int \cos 2x\sqrt{\sin 2x}\,dx$

12. $\int (x+1)\sin\left(x^2+2x+3\right)dx$

13. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt$

14. $\int x^2\sqrt{x^3+1}\,dx$

15. $\int \frac{2}{\sqrt{3x-7}}\,dx$

16. $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx$

17. $\int \frac{x}{\sqrt{x+1}}\,dx$

18. $\int x\sqrt{2x+1}\,dx$

19. $\int \sqrt{x}\sqrt{x\sqrt{x}+1}\,dx$

20. $\int x\tan\left(x^2\right)\sec\left(x^2\right)dx$

21. $\int\left(x^2+1\right)\sqrt{x-2}\,dx$

22. $\int \frac{x^2+2x}{x^2+2x+1}\,dx$

23. $\int \frac{1}{x^2+6x+9}\,dx$

24. $\int \frac{\sec^2 x}{\left(1+\tan x\right)^3}\,dx$

25. $\int \frac{\sin x}{\left(2+3\cos x\right)^2}\,dx$

26. $\int x\tan^2\left(x^2\right)\sec^2\left(x^2\right)dx$

27. $\int\left(\tan 2x+\cot 2x\right)^2\,dx$

28. $\int \frac{xe^{x^2}}{e^{x^2}+1}dx$

29. $\int \frac{1}{\sqrt{-x^2+5x-6}}dx$

30. $\int \frac{x}{1+x^2}dx$

31. $\int \frac{4}{5x\sqrt{x^2-3}}dx$

32. $\int \frac{x^2}{1+x^2}dx$

33. $\int xe^{x^2}dx$

34. $\int \frac{x}{\sqrt{x-1}}dx$

35. $\int \left(6x+\frac{7}{\sqrt{9-x^2}}\right)dx$

36. $\int x^2\sqrt{x+1}dx$

37. $\int \left(1+e^{-x}\right)^2 dx$

38. $\int \frac{6\cos x-2\sin x}{6\sin x+2\cos x}dx$

39. $\int \frac{4}{x}\sqrt[3]{\left(1+2\ln x\right)^2}dx$

40. $\int \frac{2e^{\tan x}+5}{\cos^2 x}dx$

41. $\int \frac{\left(1-x^2\right)^{-1/2}}{3+2\arcsin x}dx$

42. $\int \frac{t^3}{\sqrt{1-t^8}}dt$

43. $\int \frac{5-x}{\sqrt{4-5x^2}}dx$

Part III: Solve the differential equations. If no initial value is indicated, find the general solution.

44. Find the value of $y\left(\frac{5\pi}{3}\right)$ when $\frac{dy}{d\theta} = \cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)$ and $y(0) = 0$.

45. Find the value of $y(\pi)$ when $\frac{dy}{dx} = 8e^{-2x} - 2\sin x$ and $y(0) = 4$

46. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$ and $f(0) = 1$.

47. $\frac{dy}{dt} = (t+1)e^{\frac{5}{2}t^2+5t}$

48. $f'(x) = \frac{1+e^{3x}}{e^{3x}+3x}$

49. $y' = \frac{\sin(\ln 5x)}{x}$

50. $\frac{dy}{dx} = \frac{1}{1+9x^2}$ where $y\left(\frac{1}{3}\right) = 2$

51. $\frac{dy}{dx} = \left(1+y^2\right)\tan x$ if $y(0) = \sqrt{3}$

52. $\frac{dy}{dx} = 1$

53. $\frac{dy}{dx} - yx = 0$

54. $e^y \frac{dy}{dx} = 1$

55. $y^2x^2 \frac{dy}{dx} = x$

Part IV: Challenging ones

56. $\int \frac{7}{\sqrt{x}\sqrt{2-x}} dx$

57. $\int_0^1 \frac{x^2+4x+1}{3x^2+3} dx$

58. $\int_0^{\pi/2} \left(2\sin\theta - \sin^3\theta\right) d\theta$

59. $\int_0^{\pi/4} \frac{3\cos x - 4\sin x}{\cos^3 x}\,dx$

60. $\int 3t^3\left(t^2+4\right)^5\,dt$

61. $\int x^3\sqrt{x^2-1}\,dx$

62. $\int\left(1+e^{-x}\right)^{-1}\,dx$

63. $\int_1^e \frac{1}{x}\left[f'\left(\ln x\right)+2\right]dx$ when $f\left(0\right)=1$ and $f\left(1\right)=4$

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Mixed Integration Worksheet

Part I:
For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int (x^3+1)\,dx$
$= \frac{1}{4}x^4 + x + C$
(nothing)

$\int x^2(x^3+1)^4\,dx$
$= \frac{1}{3}\left(\frac{1}{5}\right)(x^3+1)^5 + C$
corr ↑ pwr rule
$= \frac{1}{15}(x^3+1)^5 + C$
(substitution)

$\int \sqrt{x^3+1}\,dx$
(can't do)

$\int (x^3+1)^2\,dx$
$= \int (x^6+2x^3+1)\,dx$
$= \frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$
(algebra)

B. $\int \sqrt{x}(1-x^2)\,dx$
$\int (x^{1/2} - x^{5/2})\,dx$
$\frac{2}{3}x^{3/2} - \frac{2}{7}x^{7/2} + C$
(algebra)

$\int \sqrt{1-x^2}\,dx$
(can't do)

$\int \frac{1}{\sqrt{1-x^2}}\,dx$
$= \arcsin x + C$
(nothing)

$\int \frac{x\,dx}{\sqrt{1-x^2}}$
$= \int x(1-x^2)^{-1/2}\,dx$
$\left(-\frac{1}{2}\right)(2)(1-x^2)^{1/2} + C$
$-\sqrt{1-x^2} + C$
(substitution)

C. $\int \cos^2 x \sin^3 x\,dx$
$\int \cos^2 x(\sin^2 x)\sin x\,dx$
$\int \cos^2 x \sin x(1-\cos^2 x)\,dx$
$\int (\cos^2 x \sin x - \cos^4 x \sin x)\,dx$
$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$
(trig sub/sub)

$\int \sqrt{1-\cos^2 x}\,dx$
$\int \sqrt{\sin^2 x}\,dx$
$\int \sin x\,dx$
$-\cos x + C$
(trig sub/alg)

$\int \frac{dx}{\cos^2 x}$
$\int \sec^2 x\,dx$
$\tan x + C$
(trig sub/nothing)

$\int \frac{dx}{\cos x\sqrt{\sin x}}$
(can't do)

D. $\int \tan x \sec x\,dx$
$= \sec x + C$
(nothing)

$\int \tan x \cos x\,dx$
$\int \frac{\sin x}{\cos x}\cdot\cos x\,dx$
$\int \sin x\,dx$
$-\cos x + C$
(trig id)

$\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$
$\int \sec^2 x \cdot (\tan x)^{-1/2}\,dx$
$2(\tan x)^{1/2} + C$
$2\sqrt{\tan x} + C$
(substitution)

$\int \frac{dx}{\tan x + 1}$
(can't do)

E. $\int e^{-x^2}\,dx$
(Can't do)

$\int \frac{e^x}{3+e^x}\,dx$
$\int e^x(3+e^x)^{-1}\,dx$
$\ln|3+e^x| + C$
(substitution)

$\int (e^x+3)\,dx$
$= e^x + 3x + C$
(nothing)

$\int \frac{\ln(e^{2x})}{x^2}\,dx$
$= \int \frac{2x}{x^2}\,dx$
$= \int 2\left(\frac{1}{x}\right)dx$
$= 2\ln|x| + C$
(algebra)

Part II: Evaluate the integrals

1. $\int (5x+4)^5\,dx$

$= \left(\frac{1}{5}\right)\left(\frac{1}{6}\right)(5x+4)^6 + C$

$= \frac{1}{30}(5x+4)^6 + C$

2. $\int 3t^2\left(t^3+4\right)^5\,dt$

$= \frac{1}{6}(t^3+4)^6 + C$

3. $\int \sqrt{4x-5}\,dx$

$= \left(\frac{1}{4}\right)\left(\frac{2}{3}\right)(4x-5)^{3/2} + C$

$= \frac{1}{6}(4x-5)^{3/2} + C$

4. $\int t^2\left(t^3+4\right)^{-1/2}\,dt$

$= \left(\frac{1}{3}\right)(2)(t^3+4)^{1/2} + C$

$= \frac{2}{3}\sqrt{t^3+4} + C$

5. $\int \cos(2x+1)\,dx$

$= \frac{1}{2}\sin(2x+1) + C$

6. $\int \sin^{10} x \cos x\,dx$

$\frac{1}{11}\sin^{11} x + C$

7. $\int \frac{\sin x}{\cos^5 x}\,dx$

$= \int \sin x\,(\cos x)^{-5}\,dx$

$= (-1)\left(-\frac{1}{4}\right)(\cos x)^{-4} + C$

$= \frac{1}{4\cos^4 x} + C$

8. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx$

$= 2\left(\frac{1}{3}\right)(\sqrt{x}-1)^3 + C$

$= \frac{2}{3}(\sqrt{x}-1)^3 + C$

9. $\int \sqrt{x^3+x^2}\left(3x^2+2x\right)dx$

$= \frac{2}{3}(x^3+x^2)^{3/2} + C$

10. $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx$

$= \frac{1}{2}\left(-\frac{1}{2}\right)(x^2+2x+2)^{-2} + C$

$= -\frac{1}{4(x^2+2x+2)^2} + C$

11. $\int \cos(2x)\sqrt{\sin(2x)}\,dx$

$= \frac{1}{2}\left(\frac{2}{3}\right)(\sin 2x)^{3/2} + C$

$= \frac{1}{3}(\sin 2x)^{3/2} + C$

12. $\int (x+1)\sin\left(x^2+2x+3\right)dx$

$= -\frac{1}{2}\cos(x^2+2x+3) + C$

13. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt$

$= (-1)\left(\frac{1}{4}\right)\left(1+\frac{1}{t}\right)^4 + C$

$= -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$

14. $\int x^2\sqrt{x^3+1}\,dx$

$= \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(x^3+1)^{3/2} + C$

$= \frac{2}{9}(x^3+1)^{3/2} + C$

15. $\int \frac{2}{\sqrt{3x-7}}\,dx$

$= \int 2(3x-7)^{-1/2}\,dx$

$= 2\left(\frac{1}{3}\right)(2)(3x-7)^{1/2} + C$

$= \frac{4}{3}\sqrt{3x-7} + C$

16. $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx$

$= \int \frac{1}{\sqrt{x}}(\sqrt{x}+1)^{-2}\,dx$

$= 2(-1)(\sqrt{x}+1)^{-1} + C$

$= \frac{-2}{\sqrt{x}+1} + C$

17. $\int \frac{x}{\sqrt{x+1}}\,dx$ * u-sub

$u = x+1 \quad x = u-1$

$du = dx$

$= \int (u-1)u^{-1/2}\,du$

$= \int \left(u^{1/2} - u^{-1/2}\right)du$

$= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$

18. $\int x\sqrt{2x+1}\,dx$ * u-sub

$u = 2x+1 \quad x = \frac{1}{2}(u-1)$

$du = 2dx$

$dx = \frac{1}{2}du$

$= \left(\frac{1}{2}\right)\frac{1}{2}\int (u-1)u^{1/2}\,du$

$= \frac{1}{4}\int \left(u^{3/2} - u^{1/2}\right)du$

$= \frac{1}{4}\left[\frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right] + C$

19. $\int \sqrt{x}\sqrt{x\sqrt{x}+1}\,dx$

$= \int x^{1/2}(x^{3/2}+1)^{1/2}\,dx$

$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)(x^{3/2}+1)^{3/2}+C$

$= \frac{4}{9}(x^{3/2}+1)^{3/2}+C$

20. $\int x\tan(x^2)\sec(x^2)\,dx$

$= \frac{1}{2}\sec(x^2)+C$

21. $\int (x^2+1)\sqrt{x-2}\,dx$ * u-sub $u = x-2,\ x = u+2$, $du = dx$

$= \int ((u+2)^2+1)u^{1/2}\,du$

$= \int (u^2+4u+5)u^{1/2}\,du = \int (u^{5/2}+4u^{3/2}+5u^{1/2})\,du$

$= \frac{2}{7}(x-2)^{7/2}+\frac{8}{5}(x-2)^{5/2}+\frac{10}{3}(x-2)^{3/2}+C$

22. $\int \frac{x^2+2x}{x^2+2x+1}\,dx$ * Long divide

$x^2+2x+1 \,\big|\, x^2+2x$ (quotient 1, remainder -1)

$= \int \left(1-\frac{1}{(x+1)^2}\right)dx$

$= \int (1-(x+1)^{-2})\,dx$

$= x+(x+1)^{-1}+C$

$= x+\frac{1}{x+1}+C$

23. $\int \frac{1}{x^2+6x+9}\,dx$

$= \int \frac{1}{(x+3)^2}\,dx$

$= \int (x+3)^{-2}\,dx$

$= \frac{-1}{x+3}+C$

24. $\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx$

$= \int \sec^2 x\,(1+\tan x)^{-3}\,dx$

$= -\frac{1}{2}(1+\tan x)^{-2}+C$

$= \frac{-1}{2(1+\tan x)^2}+C$

25. $\int \frac{\sin x}{(2+3\cos x)^2}\,dx$

$= \int \sin x\,(2+3\cos x)^{-2}\,dx$

$= -3(-1)(2+3\cos x)^{-1}+C$

$= \frac{3}{2+3\cos x}+C$

26. $\int x\tan^2(x^2)\sec^2(x^2)\,dx$

$= \int x\cdot\sec^2(x^2)\cdot(\tan(x^2))^2\,dx$

$= \frac{1}{2}\left(\frac{1}{3}\right)(\tan(x^2))^3+C$

$= \frac{1}{6}\tan^3(x^2)+C$

27. $\int (\tan 2x+\cot 2x)^2\,dx$

$= \int (\tan^2 2x+2\tan 2x\cdot\cot 2x+\cot^2 2x)\,dx$

$= \int (\sec^2 2x-1+2+\csc^2 2x-1)\,dx$

$= \int (\sec^2 2x+\csc^2 2x)\,dx$

$= \frac{1}{2}\tan 2x-\frac{1}{2}\cot 2x+C$

28. $\int \frac{xe^{x^2}}{e^{x^2}+1}\,dx$

$= \frac{1}{2}\ln\left|e^{x^2}+1\right|+C$

or $= \frac{1}{2}\ln(e^{x^2}+1)+C$

29. $\int \frac{1}{\sqrt{-x^2+5x-6}}\,dx$ * complete the square

$= \int \frac{1}{\sqrt{-(x^2-5x+\frac{25}{4})+\frac{25}{4}-6}}$

$= \int \frac{1}{\sqrt{\frac{1}{4}-(x-\frac{5}{2})^2}}\,dx,\ a=\frac{1}{2},\ u=x-\frac{5}{2}$

$= \arcsin\left(\frac{x-\frac{5}{2}}{\frac{1}{2}}\right)+C = \arcsin(2x-5)+C$

30. $\int \frac{x}{1+x^2}\,dx$

$= \frac{1}{2}\ln\left|1+x^2\right|+C$

or $= \frac{1}{2}\ln(1+x^2)+C$

31. $\int \frac{4}{5x\sqrt{x^2-3}}\,dx$

$= \frac{4}{5}\int \frac{4}{x\sqrt{x^2-(\sqrt{3})^2}}\,dx$

$= \frac{4}{5}\left(\frac{1}{\sqrt{3}}\right)\operatorname{arcsec}\frac{|x|}{\sqrt{3}}+C$

$= \frac{4}{5\sqrt{3}}\operatorname{arcsec}\frac{|x|}{\sqrt{3}}+C$

32. $\int \frac{x^2}{1+x^2}\,dx$ * Long divide

$x^2+1 \,\big|\, x^2$ (quotient 1, remainder -1)

$= \int \left(1-\frac{1}{x^2+1}\right)dx$

$= x-\arctan x+C$

33. $\int xe^{x^2}\,dx$

$= \frac{1}{2}e^{x^2}+C$

34. $\int \frac{x}{\sqrt{x-1}}\,dx$ * u-sub $u = x-1,\ x = u+1$, $du = dx$

$= \int (u+1)u^{-1/2}\,du$

$= \int (u^{1/2}+u^{-1/2})\,du$

$= \frac{2}{3}(x-1)^{3/2}+2(x-1)^{1/2}+C$

35. $\int \left(6x+\frac{7}{\sqrt{9-x^2}}\right)dx$

$= \int \left(6x+7\left(\frac{1}{\sqrt{3^2-x^2}}\right)\right)dx$

$= 3x^2+7\arcsin\left(\frac{x}{3}\right)+C$

36. $\int x^2\sqrt{x+1}\,dx$ * u-sub $u = x+1,\ x = u-1$, $du = dx$

$\int (u-1)^2 u^{1/2}\,du$

$\int (u^2-2u+1)u^{1/2}\,du$

$\int (u^{5/2}-2u^{3/2}+u^{1/2})\,du$

$= \frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}+C$

37. $\int (1+e^{-x})^2 dx$

$= \int (1 + 2e^{-x} + e^{-2x}) dx$

$= x - 2e^{-x} - \frac{1}{2}e^{-2x} + C$

38. $\int \frac{6\cos x - 2\sin x}{6\sin x + 2\cos x} dx$

$= \ln|6\sin x + 2\cos x| + C$

39. $\int \frac{4}{x}\sqrt[3]{(1+2\ln x)^2}\, dx$

$= 4\int \left(\frac{1}{x}\right)(1+2\ln x)^{2/3} dx$

$= 4\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)(1+2\ln x)^{5/3} + C$

$= \frac{6}{5}(1+2\ln x)^{5/3} + C$

40. $\int \frac{2e^{\tan x} + 5}{\cos^2 x} dx$

$= \int \sec^2 x (2e^{\tan x} + 5) dx$

$= \int (2\sec^2 x e^{\tan x} + 5\sec^2 x) dx$

$= 2e^{\tan x} + 5\tan x + C$

41. $\int \frac{(1-x^2)^{-1/2}}{3 + 2\arcsin x} dx$

$= \int \left(\frac{1}{\sqrt{1-x^2}}\right)\left(\frac{1}{3+2\arcsin x}\right) dx$

$= \frac{1}{2}\ln|3 + 2\arcsin x| + C$

42. $\int \frac{t^3}{\sqrt{1-t^8}} dt$

$= \int t^3 \left(\frac{1}{\sqrt{1-(t^4)^2}}\right) dt$

$= \frac{1}{4}\arcsin t^4 + C$

43. $\int \frac{5-x}{\sqrt{4-5x^2}} dx$

$= \int \left(\frac{5}{\sqrt{2^2 - (\sqrt{5}x)^2}} - x(4-5x^2)^{-1/2}\right) dx$

$= \frac{5}{\sqrt{5}}\arcsin\frac{\sqrt{5}x}{2} + \frac{1}{10}(2)(4-5x^2)^{1/2} + C$

$= \frac{5}{\sqrt{5}}\arcsin\left(\frac{\sqrt{5}}{2}x\right) + \frac{1}{5}\sqrt{4-5x^2} + C$

Part III: Solve the differential equations. If no initial value is indicated, find the general solution.

44. Find the value of $y\left(\frac{5\pi}{3}\right)$ when $\frac{dy}{d\theta} = \cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)$ and $y(0) = 0$.

$\int dy = \int \left(\cos\left(\frac{\theta}{5}\right)\right)^2 \cdot \sin\left(\frac{\theta}{5}\right) d\theta$

$y = -\frac{1}{3}(5)\cos^3\left(\frac{\theta}{5}\right) + C$

at $(0,0)$: $0 = -\frac{5}{3} + C$, $C = \frac{5}{3}$

so $y = -\frac{5}{3}\cos^3\left(\frac{\theta}{5}\right) + \frac{5}{3}$

$y\left(\frac{5\pi}{3}\right) = -\frac{5}{3}\left(\cos\left(\frac{\pi}{3}\right)\right)^3 + \frac{5}{3}$

$y\left(\frac{5\pi}{3}\right) = -\frac{5}{3}\left(\frac{1}{8}\right) + \frac{5}{3}$

$= -\frac{5}{24} + \frac{40}{24}$

$= \boxed{\frac{35}{24}}$

45. Find the value of $y(\pi)$ when $\frac{dy}{dx} = 8e^{-2x} - 2\sin x$ and $y(0) = 4$

$\int dy = \int (8e^{-2x} - 2\sin x) dx$

$y = -4e^{-2x} + 2\cos x + C$

at $(0,4)$: $4 = -4 + 2 + C$, $C = 6$

so $y = -4e^{-2x} + 2\cos x + 6$

$y(\pi) = -4e^{-2\pi} + 2\cos\pi + 6$

$y(\pi) = \boxed{-4e^{-2\pi} + 4}$

46. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$ and $f(0) = 1$.

$f(x) = -\frac{6}{4}e^{-2x^2} + C$

$f(x) = -\frac{3}{2}e^{-2x^2} + C$

at $(0,1)$: $1 = -\frac{3}{2} + C$, $C = \frac{5}{2}$

so $f(x) = -\frac{3}{2}e^{-2x^2} + \frac{5}{2}$

$f(-1) = \boxed{-\frac{3}{2}e^{-2} + \frac{5}{2}}$

47. $\frac{dy}{dt} = (t+1)e^{\frac{5}{2}t^2 + 5t}$

$\int dy = \int (t+1)e^{\frac{5}{2}t^2 + 5t} dt$

$y = \frac{1}{5}e^{\frac{5}{2}t^2 + 5t} + C$

48. $f'(x) = \frac{1 + e^{3x}}{e^{3x} + 3x}$

$f(x) = \frac{1}{3}\ln|e^{3x} + 3x| + C$

49. $y' = \frac{\sin(\ln 5x)}{x}$

$y = -\cos(\ln 5x) + C$

50. $\frac{dy}{dx}=\frac{1}{1+9x^2}$ where $y\left(\frac{1}{3}\right)=2$

$\int dy=\int\frac{1}{1+(3x)^2}dx$

$y=\frac{1}{3}\arctan(3x)+C$

at $(\frac{1}{3},2)$: $2=\frac{1}{3}\arctan 1+C$

$2=\frac{\pi}{12}+C,\ C=2-\frac{\pi}{12}$

$y=\frac{1}{3}\arctan(3x)+2-\frac{\pi}{12}$

51. $\frac{dy}{dx}=(1+y^2)\tan x$ if $y(0)=\sqrt{3}$

$\frac{1}{1+y^2}dy=\int\tan x\,dx$

$\arctan y=-\ln|\cos x|+C$

at $(0,\sqrt{3})$: $\arctan\sqrt{3}=-\ln|\cos 0|+C$

$C=\frac{\pi}{3}$ so $y=\tan(-\ln|\cos x|+\frac{\pi}{3})$

52. $\frac{dy}{dx}=1$

$\int dy=\int dx$

$y=x+C$

53. $\frac{dy}{dx}-yx=0$

$\frac{dy}{dx}=yx$

$\frac{1}{y}dy=x\,dx$

$\ln|y|=\frac{1}{2}x^2+C$

$y=\pm e^{\frac{1}{2}x^2+C}=Ce^{\frac{1}{2}x^2}$

54. $e^y\frac{dy}{dx}=1$

$\int e^y dy=\int dx$

$e^y=x+C$

$y=\ln(x+C)$

55. $y^2x^2\frac{dy}{dx}=x$

$y^2dy=\frac{1}{x}dx$

$\frac{1}{3}y^3=\ln|x|+C$

$y^3=3\ln|x|+C$

$y=\sqrt[3]{3\ln|x|+C}$

Part IV: Challenging ones

56. $\int\frac{7}{\sqrt{x}\sqrt{2-x}}dx$ * u-sub $u=\sqrt{x},\ x=u^2$, $dx=2u\,du$

$=\int\frac{7}{u\sqrt{2-u^2}}\cdot 2u\,du=14\int\frac{1}{\sqrt{(\sqrt{2})^2-u^2}}du$

$=14\arcsin\frac{u}{\sqrt{2}}+C$

$=14\arcsin\sqrt{\frac{x}{2}}+C$

57. $\int_0^1\frac{x^2+4x+1}{3x^2+3}dx$ * Long div

$x^2+3\overline{)x^2+4x+1}$, $-x^2+3$, $4x-2$

$=\frac{1}{3}\int\left(1+\frac{4x-2}{x^2+3}\right)dx$

$=\frac{1}{3}\int\left(1+4\left(\frac{x}{x^2+3}\right)-2\left(\frac{1}{x^2+3}\right)\right)dx$

$=\frac{1}{3}\left[x+2\ln|x^2+3|-2\left(\frac{1}{\sqrt{3}}\right)\arctan\frac{x}{\sqrt{3}}\right]\Big|_0^1$

$=\frac{1}{3}\left[(1+2\ln 4-\frac{2}{\sqrt{3}}(\frac{\pi}{6}))-(2\ln 3)\right]$

58. $\int_0^{\pi/2}(2\sin\theta-\sin^3\theta)d\theta$

$\int(2\sin\theta-\sin\theta(1-\cos^2\theta))d\theta$

$=\int(2\sin\theta-\sin\theta+\sin\theta(\cos\theta)^2)d\theta$

$=\int(\sin\theta+\sin\theta(\cos\theta)^2)d\theta$

$=-\cos\theta-\frac{1}{3}\cos^3\theta\Big|_0^{\pi/2}$

$(0)-(-1-\frac{1}{3})=\frac{4}{3}$

59. $\int_0^{\pi/4}\frac{3\cos x-4\sin x}{\cos^3 x}dx$

$=\int(3\frac{\cos x}{\cos^3 x}-\frac{4\sin x}{\cos^3 x})dx$

$=\int(3\sec^2 x-4\tan x\cdot\sec^2 x)dx$

$=3\tan x-2\tan^2 x\Big|_0^{\pi/4}$

$=(3-2)-(0)$

$=\boxed{1}$

60. $\int 3t^3(t^2+4)^5\,dt$ * u-sub

$u=t^2+4,\ t^2=u-4$

$du=2t\,dt,\ t^3dt=t^2\cdot t\,dt$

$t\,dt=\frac{1}{2}du$

$=\frac{3}{2}\int(u-4)u^5du$

$=\frac{3}{2}\int(u^6-4u^5)du$

$=\frac{3}{2}\left[\frac{1}{7}(t^2+4)^7-\frac{2}{3}(t^2+4)^6\right]+C$

61. $\int x^3\sqrt{x^2-1}dx$ * usub, $x^2-1=u$, $x^2=u+1$, $x\,dx=\frac{1}{2}du$,

$=\int x^2\cdot(x^2-1)^{1/2}\cdot x\,dx$

$=\frac{1}{2}\int(u+1)u^{1/2}du$

$=\frac{1}{2}\int(u^{3/2}+u^{1/2})du$

$=\frac{1}{2}\left[\frac{2}{5}(x^2-1)^{5/2}+\frac{2}{3}(x^2-1)^{3/2}\right]+C$

62. $\int(1+e^{-x})^{-1}dx$

$=\int\frac{1}{1+e^{-x}}\left(\frac{e^x}{e^x}\right)dx$

$=\int\frac{e^x}{1+e^x}dx$

$=\ln|1+e^x|+C$

63. $\int_1^e\frac{1}{x}[f'(\ln x)+2]dx$ when $f(0)=1$ and $f(1)=4$

$=\int(\frac{1}{x}f'(\ln x)+2(\frac{1}{x}))dx$

$=f(\ln x)+2\ln|x|\Big|_1^e$

$=(f(\ln e)+2\ln e)-(f(\ln 1)+2\ln 1)$

$=f(1)+2-f(0)+2$

$=4+2-1+2$

$=\boxed{7}$