Web Link

Web Link

Visual Calculus

Visual Calculus

Java Applet

Derivative of Inverse Trigonometric Functions

Derivative of the Arcsine

$$y = \sin^{-1}(x)$$
$$x = \sin(y)$$
$$\frac{d}{dx}[x] = \frac{d}{dx}[\sin(y)]$$
$$1 = \cos(y)\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{1}{\cos(y)}$$
$$= \frac{1}{\sqrt{1-[\sin(y)]^2}}$$
$$= \frac{1}{\sqrt{1-x^2}}$$

$\frac{1}{\cos(y)}$ would be adequate for the derivative of $x = \sin(y)$, but we require the derivative of $y = \sin^{-1}(x)$. Therefore, our answer must be in terms of x.

By applying similar techniques, we obtain the rules for derivatives of **Inverse Trigonometric Functions**.

Rules:

$$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\cos^{-1}(u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx},$$

$$\frac{d}{dx}\cot^{-1}(u) = \frac{-1}{1+u^2}\frac{du}{dx},$$

$$\frac{d}{dx}\sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u| > 1$$

Example

Find $\frac{dy}{dx}$ if $y = \sin^{-1}(x^3)$

Solution:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2 = \frac{3x^2}{\sqrt{1-x^6}}$$

Example

A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}(\sqrt{t})$. What is the velocity of the particle when $t = 16$?

Solution:

$$v(t) = \frac{d}{dt}\left[\tan^{-1}(\sqrt{t})\right] = \frac{1}{1+(\sqrt{t})^2} \cdot \frac{d}{dt}\left[\sqrt{t}\right]$$

$$= \frac{1}{1+t} \cdot \frac{1}{2\sqrt{t}}$$

When t=16, the velocity is:

$$v(16) = \frac{1}{1+16} \cdot \frac{1}{2\sqrt{16}} = \frac{1}{136}$$

Example

Determine the derivative of $(\sec^{-1}(x))^2$

Solution:

$$\frac{d}{dx}(\sec^{-1}(x))^2 = 2(\sec^{-1}(x))\frac{d}{dx}(\sec^{-1}(x))$$

$$= 2(\sec^{-1}(x))\frac{1}{|x|\sqrt{x^2-1}}$$

Example

Find the derivative of $f(x) = \arctan(2x^2 - x)$

Solution:

$$f'(x) = \frac{1}{1+(2x^2-x)^2}\frac{d}{dx}(2x^2-x)$$
$$= \frac{4x-1}{1+(2x^2-x)^2}$$

Example

Determine all points where the tangent line to $y = \sin^{-1}\left(\frac{1}{2+x^2}\right)$ is horizontal.

Solution:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{1}{2+x^2}\right)^2}} \cdot \frac{-2x}{(2+x^2)^2}$$

Now the derivative equals zero (**line is horizontal**) only when x=0.

We now need only the y-value.

Therefore $y = \sin^{-1}\left(\frac{1}{2+(0)^2}\right)$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{6}$$

The point $(x, y) = \left(0, \frac{\pi}{6}\right)$