Some Practice Vector Proof Problems

1. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.

2. Prove that the diagonals of a parallelogram bisect each other.

3. Prove that the diagonals of a rhombus are perpendicular. (A rhombus is a parallelogram with four congruent sides.)

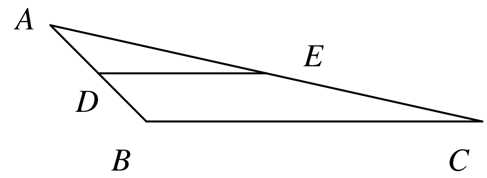
4. Prove that the diagonals of a rhombus bisect the angles of the rhombus.

5. Prove that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (This is quite useful for carpenters.)

6. Prove that the diagonals of a rectangle are congruent. (This is the converse of the statement in 5.)

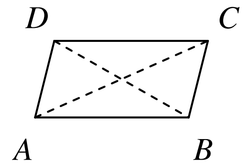
7. Let $\overline{AB}$ be a diameter of a sphere. If P is any point on the sphere other than A or B, prove that $\overline{AP}$ is orthogonal to $\overline{BP}$.

Hints:



1. Let D and E be the midpoints of $\overline{AB}$ and $\overline{AC}$.

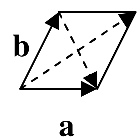
   Prove: $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$.



2. Label the four vertices of the parallelogram.

   Prove: $\frac{1}{2}\overrightarrow{AC} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DB}$

3. Draw the rhombus and label adjacent vectors **a** and **b**.

   The diagonals are **a** + **b** and **a** - **b**.

   Given: $|\mathbf{a}| = |\mathbf{b}|$

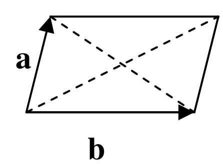
   Prove: $(\mathbf{a}+\mathbf{b}) \perp (\mathbf{a}-\mathbf{b})$

4. Same picture as in 3. Let θ_1 be the angle between **a** and **a** + **b**, and let θ_2 be the angle between **b** and **a+b**,

   Given: $|\mathbf{a}| = |\mathbf{b}|$

   Prove: $\theta_1 = \theta_2$ (Show the cosines are equal.)

5. Let **a** and **b** be as in the picture. Then

   Given: $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$

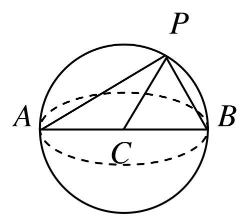
   Prove: $\mathbf{a} \perp \mathbf{b}$

6. Same picture as in 5.

   Given: $\mathbf{a} \perp \mathbf{b}$

   Prove: $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$

7. Let C be the center of the sphere.

   Given: $\overrightarrow{AC} = \overrightarrow{CB}$, $|CA| = |CP|$

   Prove: $\overline{AP} \perp \overline{BP}$